

# Uncertainty Analysis for Particulate Backscatter, Extinction and Optical Depth Retrievals reported in the CALIPSO Level 2, Version 3 Data Release.

## Introduction

The purpose of this document is to provide users of the CALIPSO level 2, version 3 data release optical property products an understanding of how the quoted uncertainties in those products are calculated. Before reading this document, readers are strongly advised to read the description of the interaction of the various algorithms that comprise the CALIPSO analysis process (the Selective Iterated Boundary Locator (SIBYL), Scene Classification Algorithms (SCA) and Hybrid Extinction Retrieval Algorithms (HERA)) provided by Winker *et al.* (2009). An understanding of the extinction retrieval process described in Young and Vaughan (2009), which details the mathematical equations used in the analysis, is also required.

Readers should note that the uncertainty analysis provided herein is different from that described in the draft CALIOP level 2 ATBD Part 4 (Young *et al.*, 2008). Although the analysis in the draft ATBD treats random and systematic errors separately, the coding and testing for that analysis is still in progress and was not ready for the version 3 data release. It is expected that it will be ready for the version 4 data release.

The uncertainty analysis described below is an earlier, simplified analysis in which it was assumed that all the uncertainties were random, uncorrelated and produced no biases. At the time of the development of this analysis, the actual magnitude (and sign) of the uncertainties in many of the parameters was largely unknown, and the simplified analysis described here was considered adequate for estimating the influences of the main uncertainties. Experience being gained with the operation of the CALIPSO instrument and in the analysis and validation of the data is providing much useful information on the uncertainties that will be used in the next data release.

## Lidar Equation

Here we follow the notation and development in Young and Vaughan (2009). For an elastic backscatter lidar system like that on board CALIPSO, the retrieval of profiles of particulate backscatter and extinction requires the solution of the two-component lidar equation:

$$P(r) = \frac{1}{r^2} E_0 \xi [\beta_M(r) + \beta_P(r)] T_M^2(0, r) T_{O_3}^2(0, r) T_P^2(0, r). \quad (1)$$

Here

$P(r)$  is the detected backscattered signal from range  $r$  from the lidar;

$\xi$  is the lidar system parameter (see CALIOP Level 1 ATBD (Hostetler *et al.*, 2008) section 3.1.2), where  $\xi = G_A C$ , and  $G_A$  is the amplifier gain and  $C$  is the lidar calibration factor;

$E_0$  is the (average) laser energy for a single or average profile;

$\beta_M(r)$  is the molecular volume backscatter coefficient, and is proportional to the molecular number density profile;

$$T_M^2(0, r) = \exp \left[ -2 \int_0^r \sigma_M(r') dr' \right] \quad (2)$$

is the profile of the two-way molecular transmittance between the lidar and range  $r$ ;

$\sigma_M(r) = S_M \beta_M(r)$  is the molecular volume extinction coefficient, where  $S_M$  is the molecular extinction-to-backscatter (or lidar) ratio;

$T_{O_3}^2(0, r) = \exp \left[ -2 \int_0^r \alpha_{O_3}(r') dr' \right]$  is the two-way ozone transmittance, where

$\alpha_{O_3}(r')$  is the ozone absorption coefficient;

$\beta_P(r)$  is the particulate volume backscatter coefficient;

$$T_P^2(0, r) = \exp \left[ -2 \int_0^r \eta(r') \tau_P(r') dr' \right] \quad (3)$$

is the particulate two-way transmittance, where

$$\tau_P(0, r) = \int_0^r \sigma_P(r') dr' = S_P \int_0^r \beta_P(r') dr' \quad (4)$$

is the particulate optical thickness (depth);

$$\sigma_P(r) = S_P \beta_P(r) \quad (5)$$

is the particulate volume extinction coefficient, where

$S_P$  is the particulate extinction-to-backscatter (or lidar) ratio and

$\eta(r)$  is a parameterization describing multiple scattering by particles. For the version 3 analysis, however, because insufficient, validated data are available on the range dependence of this quantity, a constant value of  $\eta$  is used throughout any feature so that  $\eta(r) = \eta$  for all  $r$ . This simplification is used in the following equations.

### *Modification of the attenuated backscatter coefficients for use in extinction processing*

During level 1B processing of the data, a lidar calibration factor is calculated (Hostetler *et al.*, 2008) after any signal offset has been removed. To initiate the level 2 processing, various numbers of profiles are averaged and the resulting signal processed to extract information on any detected features. CALIOP data are processed in blocks of 240 single-shot profiles, corresponding to 80 km of along-track distance. Extinction retrievals are performed on horizontal resolutions 5 km, 20 km and 80 km. Sixteen profiles, averaged to 5-km horizontal

resolution, are supplied to the HERA for each 80-km block of data. These profiles are of the attenuated backscatter signal *corrected for ozone transmittance*:

$$\beta'(0, r) = [\beta_M(r) + \beta_P(r)] T_M^2(0, r) T_P^2(0, r) = \frac{P(r)r^2}{E_0 G_A C T_{O_3}^2}. \quad (6)$$

The uncertainty in this quantity,  $\Delta\beta'$ , is also supplied. The SCA also supplies values of the multiple scatter function,  $\eta$ . Ideally a value of its uncertainty would also be supplied, but insufficient data are available on this quantity. For the version 3 data release, this uncertainty is considered to be incorporated into the uncertainty in the lidar ratio. The molecular backscatter and transmittance profiles, ozone transmittance profiles and all their uncertainties are provided by the Meteorological Manager Module (Met Manager) using profiles of molecular density and ozone absorption obtained from NASA's Global Modeling and Assimilation Office (GMAO; Bloom *et al.*, 2005). Statistics of each layer detected by the SIBYL and SCA, including the altitude of the base and top, optical thickness and lidar ratio, and their uncertainties are also supplied to the HERA.

## Uncertainty Analysis

In the following uncertainty analysis, which was used in the creation of the version 3 data products, it will be assumed that uncertainties in the different quantities are random and uncorrelated. While this is not always strictly true, it is expected that most errors incurred in this approximation will be smaller than the main contributors to the overall uncertainties. The results of this simplified analysis are expected to give acceptable estimates of the uncertainties for low to moderate optical depths.

Following the development given in Young and Vaughan (2009), the solution for the particulate backscatter coefficient at each range bin within a feature is given by iterating between the equations

$$\beta_P(r) = \frac{\beta'_N(r)}{T_M^2(r_N, r) T_P^2(r_N, r)} - \beta_M(r), \quad (7)$$

and

$$T_P^2(r_N, r) = \exp \left[ -2\eta S_P \int_{r_N}^r \beta_P(z) dz \right] = \exp[-2\eta \tau_P(r_N, r)]. \quad (8)$$

Here,  $\beta'_N(r)$ , is the attenuated backscatter after normalization by dividing by the transmittance losses between the lidar and a normalization range that is defined so that  $0 < r_N < r$ .

The uncertainty in the retrieved particulate backscatter coefficient can be written

$$(\Delta\beta_p(r))^2 = \beta_T^2(r) \left[ \left( \frac{\Delta\beta'_N(r)}{\beta'_N(r)} \right)^2 + \left( \frac{\Delta T_M^2(r_N, r)}{T_M^2(r_N, r)} \right)^2 + \left( \frac{\Delta T_P^2(r_N, r)}{T_P^2(r_N, r)} \right)^2 \right] + (\Delta\beta_M(r))^2. \quad (9)$$

Here,  $\beta_T(r)$  is the total backscatter coefficient at range  $r$ . The uncertainty in the particulate transmittance term in equation (9) is derived from equation (8):

$$\left( \Delta T_P^2(r_N, r) \right)^2 = \left( \frac{\partial T_P^2(r_N, r)}{\partial \eta} \right)^2 (\Delta\eta)^2 + \left( \frac{\partial T_P^2(r_N, r)}{\partial \tau_p(r_N, r)} \right)^2 (\Delta\tau_p(r_N, r))^2, \quad (10)$$

which leads to

$$\left( \frac{\Delta T_P^2(r_N, r)}{T_P^2(r_N, r)} \right)^2 = (2\tau_p(r_N, r)\Delta\eta)^2 + (\eta(r))^2 (2\Delta\tau_p(r_N, r))^2. \quad (11)$$

However, as explained above in the discussion of the notation for the lidar equation, the uncertainty in  $\eta$  is set to zero for this data release, hence the first terms in both (10) and (11) are also zero.

The optical thickness can be expressed as the product of the lidar ratio and the integrated backscatter:

$$\tau_p(r_N, r) = S_p \gamma_p(r_N, r), \quad (12)$$

from which we can derive an expression for the uncertainty:

$$(\Delta\tau_p(r_N, r))^2 = (\Delta S_p)^2 \gamma_p^2(r_N, r) + (\Delta\gamma_p(r_N, r))^2 S_p^2 + 2\tau_p(r_N, r) \text{co}(\Delta S_p, \gamma_p). \quad (13)$$

However, for the version 3 data release, values of  $\Delta S_p$  are still being evaluated and have large uncertainties. The values of the covariances in the third term in (13) are even less well known at this stage and have been set to zero. Hence, quoted uncertainties are likely to be underestimated, particularly for larger values of optical depth. In the software implementation of the extinction retrieval algorithm, the integrated particulate backscatter coefficient is calculated at each range increment using trapezoidal integration:

$$\gamma_p(r_N, r) = \gamma_{P_j} = 0.5 \sum_{i=j_{cal}}^{j-1} (\beta_{P_i} + \beta_{P_{i+1}})(r_{i+1} - r_i) = 0.5 \sum_{i=j_{cal}}^{j-1} (\beta_{P_i} + \beta_{P_{i+1}})\delta r_{i+1}, \quad (14)$$

where  $\delta r_j$  is the  $j$ th range increment. (These range increments are not all the same.) The uncertainty in  $\gamma_{P_j}$  can now be calculated as

$$(\Delta\gamma_{P_j})^2 = 0.25((\delta r_j \Delta\beta_{P_j})^2 + (\delta r_{j_{cal}} \Delta\beta_{P_{j_{cal}}})^2 + \sum_{i=j_{cal}+1}^{j-1} (\delta r_i + \delta r_{i+1})^2 (\Delta\beta_{P_i})^2). \quad (15)$$

However, as the first term in equation (15) is the unknown in equation (9), this term is evaluated by initially setting  $\Delta\beta_j$  to  $\Delta\beta_{j-1}$  and iterating using equations (9) and (15) until convergence is achieved following a similar procedure to that used in the iteration of equations (7) and (8). Finally, the uncertainty in the particulate extinction coefficient can be derived from equation (5):

$$\Delta\sigma_p(r) = \left( \left( \frac{\Delta S_p}{S_p} \right)^2 + \left( \frac{\Delta\beta_p(r)}{\beta_p(r)} \right)^2 \right)^{1/2} \sigma_p(r). \quad (16)$$

Note that, although we could calculate the uncertainty in the retrieved particulate optical thickness by integrating (16) over the depth of the feature, this would result in a significant underestimation because of the way the systematic error in  $S_p$  is combined with the random error,  $\Delta\beta_p$ , at each range step. Use of (13) largely overcomes this problem.

## References

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